The Coupling of Angular Momenta in Nuclear Reactions*

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Seven nuclear reactions have been examined where the angular distribution of reaction products depends on the type of coupling of angular momentum vectors in the nucleus. In six of the seven, the experimental angular distribution is consistent with the assumption of Russell-Saunders coupling in the nucleus. In three of the seven, the experiments are consistent with the assumption that the bombarding particle brings a definite j in the reaction. Neither assumption is sufficient to explain the result in one reaction ($B^{11}+P$).

I. INTRODUCTION

CONSIDERABLE use has been made, in nuclear reactions, of angular distributions and angular correlations to establish the total angular momenta and parities of nuclear states. In some cases, however, such determinations are hampered by the appearance of parameters in the calculated angular distribution which, for definite total angular momentum and parity, lead to a continuous range of possible angular distributions or angular correlations. In fact, such undetermined angular distributions are found whenever three (or more) nonzero angular momenta must be added to give a fourth and the resultant can be reached in more than one way. Geometrically we may think of a four-sided polygon with given lengths for the sides: if the lengths are such that the structure is flexible, then the angular distribution will be incompletely determined by the given angular momenta. In a typical example, the three angular momenta may be the angular momentum of the target nucleus, the intrinsic angular momentum of the bombarding particle, and the orbital angular momentum of the bombarding particle; these must add together to give the angular momentum of the (resonant) compound nucleus.

Where possible, it is customary to combine the several angular momenta in such a way that the result can be expressed as an incoherent mixture (with arbitrary relative weight) of two distribution functions which then represent the extreme possible distributions. Thus, if the arbitrariness arises from nonzero intrinsic angular momenta (J) of bombarding particle and target nucleus and nonzero orbital angular momentum (l) of bombarding particle in the angular distribution of a resonance reaction using unpolarized beam and target, then the intrinsic J's of bombarding particle J_1 and target nucleus J_0 may be added to form a "channel spin" J_c: $\mathbf{J}_0+\mathbf{J}_1=\mathbf{J}_c$, where $|J_0-J_1|\leq J_c\leq J_0+J_1$. The angular distribution will then have an arbitrary parameter if the J of the compound nucleus can be formed by adding l to more than one of the possible values of J_c. The angular distribution can be calculated uniquely for each J_c and the various results combined with arbitrary weight. Since the calculation involves summing the squared amplitudes for the spin states of both particles, one may add the spins first to form the "channel spin" and then sum the squared amplitudes of the states of this quantity with no interference between different channel spins. The same procedure can be used when the residual nucleus and particle have nonzero J and l; a "channel spin" J_c' may be formed out of the J_3 of the residual nucleus and J_2 of the emitted particle and the various distribution functions calculated for each J_c' and combined with arbitrary relative weight.

In the case of electromagnetic radiation, where such ambiguity is also possible, as in cases where both electric quadrupole and magnetic dipole radiation are permitted, it is customary to allow for the arbitrariness by means of a coherent mixture described by an amplitude mixture with an arbitrary complex parameter. In the case of an ambiguous angular correlation arising from emission of particles with intrinsic angular momenta there is no advantage in the channel spin method; it is usually most convenient to add the l and intrinsic angular momentum of the emitted particle to form several j values which must then be combined coherently with arbitrary relative amplitude and phase to describe all possible results.

This arbitrariness in a calculated distribution function is, of course, just a result of our ignorance of the nuclear wave functions. The results would be uniquely determined by complete knowledge of the wave functions involved. It follows that, if we know the J's and l appropriate to a certain reaction, the actual observed angular distribution, if not uniquely calculated, tells us additional facts concerning the nuclear wave functions. The assumption of Russell-Saunders coupling (i.e., that states are describable by L, S, and J, each conserved) is usually sufficient to resolve the ambiguity and give unique predictions of angular distributions for definite L, S, and J. Or, again, the assumption that a bombarding (or emitted) proton or neutron interacts with a single definite j out of the two possible values $l\pm\frac{1}{2}$, instead of a coherent superposition of the two, is sufficient to lead to unique angular distributions. These two assumptions are, of course, closely related to possible nuclear models, i.e., the Russell-Saunders coupling shell model and the $j-j$ coupling shell model. It is thus of interest to examine whether the results of angular distribution and angular correlation experi-

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ments, which are left uncertain by knowledge of the J's involved, agree with either the $L-S$ or the $j-j$ calculations.

II. EXPERIMENTAL EVIDENCE ON ANGULAR DISTRIBUTIONS

A. $T(p,\gamma)He^4$

The radiation is quite accurately $\sin^2\theta$.[1] It is assumed that a single (broad) state of He^{4*} is involved, although examination of the γ-yield itself is not sufficient to establish this fact (examination of the neutron yield lends some support to this view; analysis of the scattering phases should be more decisive). The radiation can be described as due to p-wave protons ($l=1$) forming a $J=1^-$ state in He^4 which radiates by electric dipole to the ground state of He^4.

"Channel spin" calculation

Channel spin J_c	γ-ray angular distribution
0	$\sin^2\theta$
1	$1+\cos^2\theta$

The general distribution is $1+A\cos^2\theta$, where $-1\leq A \leq 1$. The experimental result corresponds to $J_c=0$ with zero admixture of $J_c=1$.

$j-j$ calculation

Incoming proton	γ-ray angular distribution
$p_{\frac{1}{2}}$	spherical—a general result for $j=\frac{1}{2}$
$p_{\frac{3}{2}}$	$1-\frac{3}{5}\cos^2\theta$

$L-S$ calculation

Triton ground state	Compound state	Angular distribution
${}^2S_{\frac{1}{2}}$	${}^1P_1$	$1-\cos^2\theta$
${}^2S_{\frac{1}{2}}$	${}^3P_1$	$1+\cos^2\theta$

We see that the observed angular distribution can not be described by the capture of either a $p_{1/2}$ or $p_{3/2}$ proton. On the other hand, if the assumption of resonance is valid, it is consistent with a $L-S$ description where the ground state of T is ${}^2S_{1/2}$ and the compound state of He^4 is ${}^1P_1$.

B. $Li^7(p,\gamma)Be^8$—440-Kev Resonance

The requirements of spherical symmetry[2] for the γ-rays, and p-wave protons to explain the scattering,[3] can, as Devons[2] has pointed out, be reconciled with p-wave protons forming a compound state in Be^8 with $J=1^+$ and a certain mixture of channel spins 1 and 2 in the ratio 1 to 5. This mixture of channel spins results in fact in equal population of the magnetic substates of the compound state so that all processes will proceed with spherical symmetry. The question here is, does this spherical symmetry (which appears accidental from the point of view of channel spin) arise naturally in some other description such as $j-j$ or $L-S$. In fact, it does. It is easy to see that the assumption that the proton is $p_{1/2}$ also leads to spherical symmetry since $j=\frac{1}{2}$ cannot lead to any $\cos^2\theta$ terms. On the $L-S$ description, the ground state of Li^7 is ${}^2P_{3/2}$ so that the compound state could be ${}^1P_1$, ${}^3S_1$, ${}^3P_1$, or ${}^3D_1$. Of these, the ${}^3S_1$ again leads to equal population of the substates of the compound state and to spherical symmetry. Thus the observations are consistent either with the assumption that only $p_{1/2}$ protons interact or with an $L-S$ description where the resonance is ${}^3S_1$.

C. $Li^7(t,\alpha)He^6$—840-Kev Resonance in Be^{10} Observed with 240-Kev Tritons

The α's leaving He^6 in the ground state are distributed as $1-A\cos^2\theta$ with $A\sim 1$, whereas the α's leaving He^6 in its 1.7-Mev excited state are nearly spherical.[4]

It seems reasonable to ascribe this resonance to p-wave tritons on Li^7 forming a compound state of Be^{10} with $J=2$, even, which emits d-wave α's to the ground state of He^6 and (predominantly) s-wave α's to the excited state of He^6, assumed to be $J=2$, even. On this basis we make the following calculations.

The spherical symmetry of the α's to the excited state follows immediately from the s-wave assignment. For the ground-state α's we have the following results:

Channel spin calculation

"Channel spin"	Angular distribution
1	$1+3\cos^2\theta$
2	$1-\cos^2\theta$

The general angular distribution is $1+A\cos^2\theta$, where $-1\leq A\leq 3$.

$L-S$ calculation

We take the ground state of Li^7 as ${}^2P_{3/2}$ and the triton as ${}^2S_{1/2}$; then

Compound state	Angular distribution
${}^1D_2$	$1+3\cos^2\theta$
${}^3D_2$	1
${}^3P_2$	$1-6/7\cos^2\theta$

The $j-j$ calculation has no special significance since the triton is not a single particle. It would give the following:

Incoming triton	Angular distribution
$p_{\frac{1}{2}}$	spherical
$p_{\frac{3}{2}}$	spherical

The observations are thus consistent with an $L-S$ description where the compound state is ${}^3P_2$.

[1] Argo, Gittings, Hemmendinger, Jarvis, Mayer, and Taschek, Phys. Rev. **76**, 182 (1949); Argo, Gittings, Hemmendinger, Jarvis, and Taschek, Phys. Rev. **78**, 691 (1950); J. Perry and S. Bame, private communication.

[2] S. Devons and G. R. Lindsey, Proc. Phys. Soc. (London) **A63**, 1202 (1950); M. B. Sterns and B. D. McDaniel, Phys. Rev. **82**, 450 (1951); Nabholtz, Stoll, and Wäffler, Helv. Phys. Acta **25**, 153 (1952).

[3] Brown, Snyder, Fowler, and Lauritsen, Phys. Rev. **82**, 159 (1951); W. D. Warters and E. A. Milne, Phys. Rev. **85**, 761 (1952); E. R. Cohen, Phys. Rev. **75**, 1463 (1949); Ph.D. thesis, California Institute of Technology, 1949 (unpublished); D. Liberman, private communication.

[4] Pepper, Almqvist, and Lorrain, Phys. Rev. **86**, 630 (1952).

D. $B^{11}(p,\gamma)C^{12}$; $B^{11}(p,\alpha)Be^8$—Resonance at 162 Kev

The experimental facts here are not entirely clear. For many years there have existed contradictory statements as to which groups of α's and which γ's were resonant at 162 kev, and as to whether or not they showed a nonspherical distribution. We will assume that the long-range α's leaving Be^8 in the ground state are resonant and are distributed as $1+0.7\cos^2\theta$.[5] We also assume that the 11-Mev γ-ray leaving C^{12} in its 4-Mev excited state is resonant and is distributed as $1+0.23\cos^2\theta$.[6] It has been shown[6] that the angular correlation of the cascade γ's and the angular distribution of the 11-Mev γ are consistent with the assumption that p-wave protons are responsible for a $J=2$, even, resonance in C^{12} which emits 11-Mev dipole (magnetic) radiation followed by 4-Mev quadrupole (electric) radiation. (However, another γ-ray angular correlation experiment seems inconsistent with the α-particle results.)[7] Unexplained is the angular distribution[5] of the short-range α's to Be^8 at 3 Mev, which are also resonant.[8] These should be largely spherical since a $J=2$, even, state of Be^8 could be reached by s-wave α's. Also unexplained is the reported[6] variation with energy of the coefficient of $\cos^2\theta$ for the 11-Mev γ-ray and the apparent absence of the 16-Mev γ-ray at resonance.

Channel spin calculation

Channel spin	11-Mev γ-distribution	Long-range α-distribution
1	$11/4+(7/4)\cos^2\theta$	$1+3\cos^2\theta$
2	$47/12-(21/12)\cos^2\theta$	$3-3\cos^2\theta$
$(1)+(3/7)(2)$	$4.4(1+0.23\cos^2\theta)$	$2.3(1+0.75\cos^2\theta)$.

$j-j$ calculation

Incoming proton	Angular distributions
$p_{\frac{1}{2}}$	spherical
$p_{\frac{3}{2}}$	spherical

$L-S$ calculation

B^{11} ground state	Compound state	Coefficient t of channel spin 2 to add to channel spin 1
$^2P_{\frac{3}{2}}$	$^1D_2$	0
$^2P_{\frac{3}{2}}$	$^3D_2$	1
$^2P_{\frac{3}{2}}$	$^3P_2$	9
$^2D_{\frac{3}{2}}$	$^1D_2$	∞
$^2D_{\frac{3}{2}}$	$^3P_2$	9
$^2D_{\frac{3}{2}}$	$^3D_2$	25/9
$^2D_{\frac{3}{2}}$	$^3F_2$	1/9

None of the above agree with the apparently correct choice of mixing ratio, $\sim 3/7$, of the two channel spins. This may just be normal or it may be associated with the fact that B^{11} lies almost at the middle of the p shell where conflict between the $j-j$ and $L-S$ coupling models can exist.

E. $C^{13}(p,\gamma)N^{14}$—1.76-Mev Resonance

The prominent γ-ray resonance at 1.76 Mev has been shown[9] to decay predominantly by radiation to the ground state of N^{14}. The angular distribution $1-0.48\cos^2\theta$ of these γ-rays is consistent[10] with the assumption of a $J=2$ odd resonance made by d-wave protons and emitting (electric) dipole radiation to the ground state of N^{14}.

Channel spin calculation

Channel spin	Angular distribution
0	$1-0.6\cos^2\theta$
1	$1-\frac{1}{3}\cos^2\theta$

The general angular distribution is $1-A\cos^2\theta$, $\frac{1}{3}\leq A\leq\frac{3}{5}$.

$j-j$ calculation

Incoming proton	Angular distribution
$d_{5/2}$	$1-0.5\cos^2\theta$

$L-S$ calculation

C^{13} ground state	Compound state	Angular distribution
$^2P_{\frac{1}{2}}$	$^3F_2$	$1-0.5\cos^2\theta$

Here the observed angular distribution is consistent with either a $d_{5/2}$ incoming proton or a $^3F_2$ compound state, although the small range allowed to A makes the result rather insensitive.

F. $N^{14}(\alpha,p)O^{17}$

Proton groups leave O^{17} in the ground ($J=5/2$, even) and first excited ($J=1/2$, even) states. It has been reported[11] that at each of two resonances at $E_\alpha=3.6$ and 4.2 Mev the angular distribution of the long-range protons is $\sim 1-\cos^2\theta$, whereas the short-range ones are spherically symmetric. In this case the arbitrariness appears in the decay of the compound nucleus. Unfortunately, no simple choice of angular momentum and parity of the compound state agrees well enough with the reported facts to warrant examination of various coupling schemes.

G. $Li^7(d,n)Be^{8*}(\gamma)Be^8$

In this reaction, a nonspherical angular correlation of neutron and γ-ray has been reported.[12] Here arbitrariness may appear in the correlation function because of the spin of the neutron. Because of this arbitrariness, the reported assignments of angular momenta in the reaction must be doubted, and there remains insufficient certainty about the various angular momenta to warrant further investigation of coupling schemes.

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[5] Haxby, Allen, and Williams, Phys. Rev. **55**, 140 (1939); Thomson, Cohen, French, and Hutchinson, Proc. Phys. Soc. (London) **A65**, 745 (1952). J. R. Oppenheimer and R. Serber, Phys. Rev. **53**, 636 (1938).
[6] Hubbard, Nelson, and Jacobs, Phys. Rev. **87**, 378 (1952).
[7] G. M. Lewis, Phil. Mag. **43**, 690 (1952).
[8] W. Whaling and W. Wenzel (private communication).
[9] Woodbury, Day, and Tollestrup, Phys. Rev. **85**, 760 (1952).
[10] R. B. Day, thesis, California Institute of Technology, 1952 (unpublished).
[11] R. R. Roy, Phys. Rev. **82**, 227 (1951).
[12] J. Thirion, Compt. rend. **233**, 37 (1951).

H. $N^{15}(p,\alpha)C^{12*}(\gamma)C^{12}$

The resonances at $E_p=429$ and 898 kev show[13] angular distributions of γ-rays and α-particles that are consistent with $J_{\text{oxygen}}=2$, odd, formed by d-wave protons emitting p-wave α-particles to an excited state of C^{12} with $J=2$, even, which decays by quadrupole (electric) γ-rays to the ground state of C^{12}.

The angular distributions at $E_p=429$ kev require a mixing ratio of (channel spin 1)$+t\times$(channel spin 0) with $t\approx 5$. The angular distributions at $E_p=898$ kev require $t=3/2$. On $j-j$ coupling we get $t=2/3$ for $d_{3/2}$ protons and $t=3/2$ for $d_{5/2}$ protons. On Russell-Saunders coupling with ${}^2P_{1/2}$ for N^{15} the various possible compound states of O^{16} give the following mixing ratios: ${}^3D_2$, $t=6$; ${}^3F_2$, $t=3/2$; ${}^3P_2$, $t=2/3$; and ${}^1D_2$, $t=0$. The 898-kev resonance is consistent with either a $d_{5/2}$ incoming proton or a ${}^3F_2$ compound state in O^{16}. The 429-kev resonance is consistent only with a ${}^3D_2$ compound state in O^{16}.

[13] Kraus, French, Fowler, and Lauritsen, Phys. Rev. **89**, 299 (1953). Mr. Kraus has also calculated the angular distributions for the reaction.

III. CONCLUSIONS

These calculations show that in most of the cases which have been examined the observed angular distributions are consistent with the assumption that the states of light nuclei belong predominantly to a definite Russell-Saunders designation. Admixtures of up to 10 percent of other designations could not be excluded in many examples, and even larger admixtures could be allowed if they led to vanishing matrix elements in the examples considered. Thus the examples considered provide no positive proof of the validity of Russell-Saunders coupling. Nevertheless, further evidence of this kind may afford an important guide to the type of coupling prevailing in nuclei.

The agreement of the angular distributions with the demands of $j-j$ coupling are somewhat less satisfactory than with $L-S$ coupling. On the other hand, the restrictions of a literal $j-j$ model are much more severe than those of Russell-Saunders coupling.

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Multiple Meson Production by High Energy Nucleon-Nucleon Collisions

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A comparison is made between Fermi's theory and Lewis, Oppenheimer, and Wouthuysen's theory of multiple meson production in high energy nucleon-nucleon collisions, and an analysis is made of some typical examples. The main difference between the two theories concerns the Lorentz covariance of the matrix element which is related, in Fermi's theory, to the Lorentz contraction of the volume in which the thermal equilibrium of virtual mesons is supposed to be established, and to the Lorentz invariant phase volume in L.O.W.'s theory. It is reasonable to take the contracted diameter of the volume in Fermi's theory to be of the order of the wavelength of the emitted meson (not that of the incident nucleon), and as a result both theories can explain to some extent the multiplicity and angular distribution. The main difference lies in the average energy of the emitted mesons in the center-of-mass system as a function of multiplicity.

I. INTRODUCTION

In recent experiments with photographic emulsions[1] at high altitude, some direct evidence was obtained for multiple meson production by the impact of very high energy nucleons. Most events are nucleon-nucleus collisions which were accompanied not only by shower particles but also by gray and black tracks. In these cases, mesons may be produced not only multiply but also plurally, as the angular distribution of the emitted mesons is sometimes much larger than can be accounted for in a single event. (The half-angle of the emitted mesons in one collision in the laboratory system is proportional to the square root of M/E^*, where E^* and M are the energy and mass, respectively, of the incident nucleon before collision.)

These events often are so complex that it is hard to disentangle from them in any unique way the criteria for truly multiple meson production. There is some direct evidence for multiple meson production by either nucleon-nucleon collision or by the collision of a nucleon at the edge of a nucleus which does not disturb the remaining nucleus. In this paper, we shall analyze the multiplicity and angular distribution and the average energy of the emitted mesons in the center-of-mass system for the latter case. There are two typical treatments of the multiple meson production by high energy nucleon-nucleon collisions. One was proposed by Lewis, Oppenheimer, and Wouthuysen (L.O.W.)[2] using

[1] Camerini, Fowler, Lock, and Muirhead, Phil. Mag. **41**, 413 (1950); Lord, Fainberg, and Schein, Phys. Rev. **80**, 970 (1950); Hopper, Biswas, and Darby, Phys. Rev. **84**, 457 (1951); E. Pickup and L. Voyvodic, Phys. Rev. **84**, 1190 (1951).

[2] Lewis, Oppenheimer, and Wouthuysen, Phys. Rev. **73**, 127 (1948).